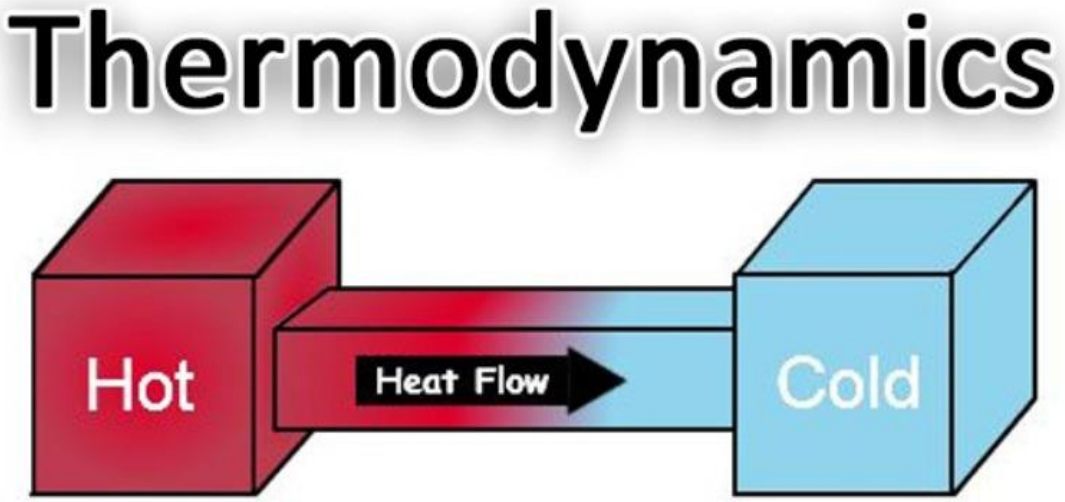
DEPARTMENT OF MECHANICAL ENGINEERING

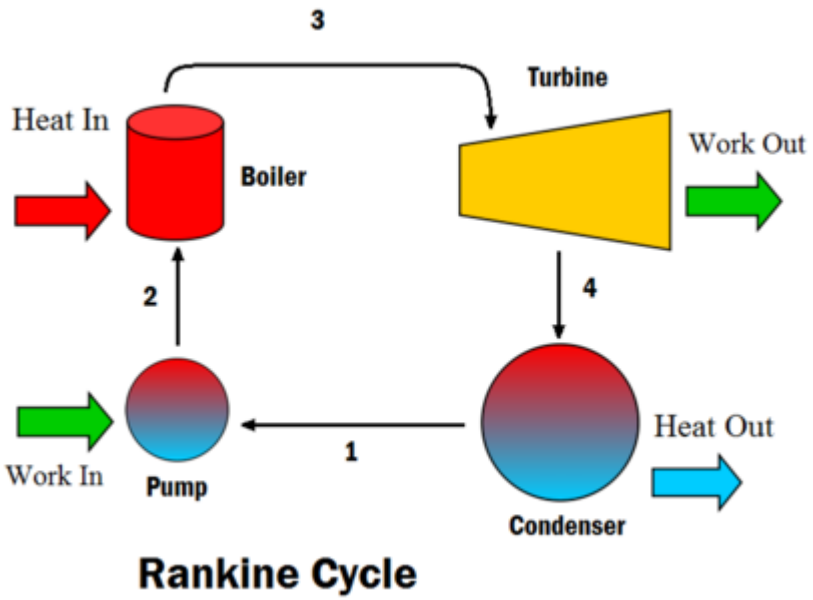
ABIT

ENGINEERING THERMODYNAMICS



Module- III

Vapour Power Cycles



Rankine Cycle

CARNOT CYCLE

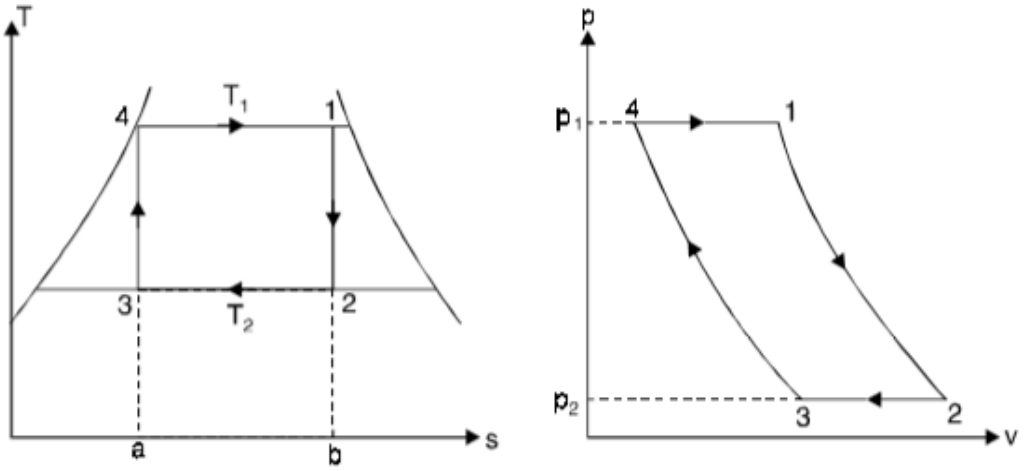
- Figure shows a Carnot cycle on T-s and p-V diagrams. It consists of (i) two constant pressure operations (4-1) and (2-3) and (ii) two frictionless adiabatics (1-2) and (3-4). These operations are discussed below :

1. Operation (4-1). 1 kg of boiling water at temperature T_1 is heated to form wet steam of dryness fraction x_1. Thus heat is absorbed at constant temperature T_1 and pressure p_1 during this operation.

2. Operation (1-2). During this operation steam is expanded isentropically to temperature T_2 and pressure p_2. The point '2' represents the condition of steam after expansion.

3. Operation (2-3). During this operation heat is rejected at constant pressure p_2 and temperature T_2. As the steam is exhausted it becomes wetter and cooled from 2 to 3.

4. Operation (3-4). In this operation the wet steam at '3' is compressed isentropically till the steam regains its original state of temperature T_1 and pressure p_1. Thus cycle is completed. Refer T-s diagram : Heat supplied at constant temperature T_1 [operation (4-1)] = area 4-1-b-a = $T_1 (s_1 - s_4)$ or $T_1 (s_2 - s_3)$.



Heat rejected at constant temperature T_2 (operation 2-3) = area 2-3-a-b = $T_2 (s_2 - s_3)$.

Since there is no exchange of heat during isentropic operations (1-2) and (3-4)

Net work done = Heat supplied – heat rejected = $T_1 (s_2 - s_3) - T_2 (s_2 - s_3) = (T_1 - T_2)(s_2 - s_3)$.

$$CARNOTCYCLE, \eta = \frac{Workdone}{Heat\ \sup\ plied} = \frac{(T_1 - T_2)(S_2 - S_3)}{T_1(S_2 - S_3)} = \frac{T_1 - T_2}{T_1}$$

Limitations of Carnot Cycle

Though Carnot cycle is simple (thermodynamically) and has the highest thermal efficiency for given values of T_1 and T_2, yet it is extremely difficult to operate in practice because of the following reasons:

1. It is difficult to compress a wet vapour isentropically to the saturated state as required by the process 3-4.

2. It is difficult to control the quality of the condensate coming out of the condenser so that the state '3' is exactly obtained.

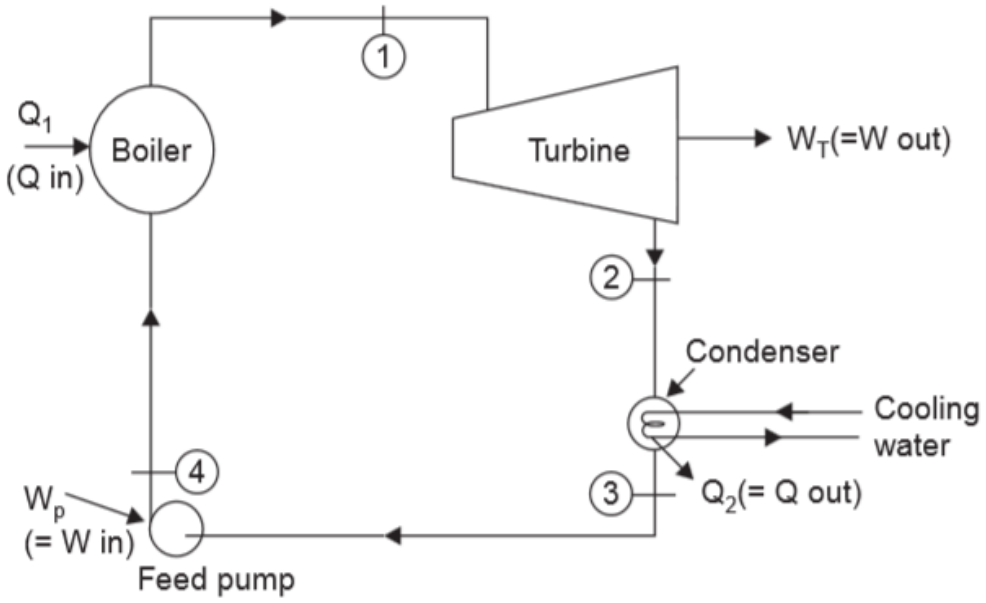
3. The efficiency of the Carnot cycle is greatly affected by the temperature T_1 at which heat is transferred to the working fluid. Since the critical temperature for steam is only 374°C, therefore, if the cycle is to be operated in the wet region, the maximum possible temperature is severely limited.

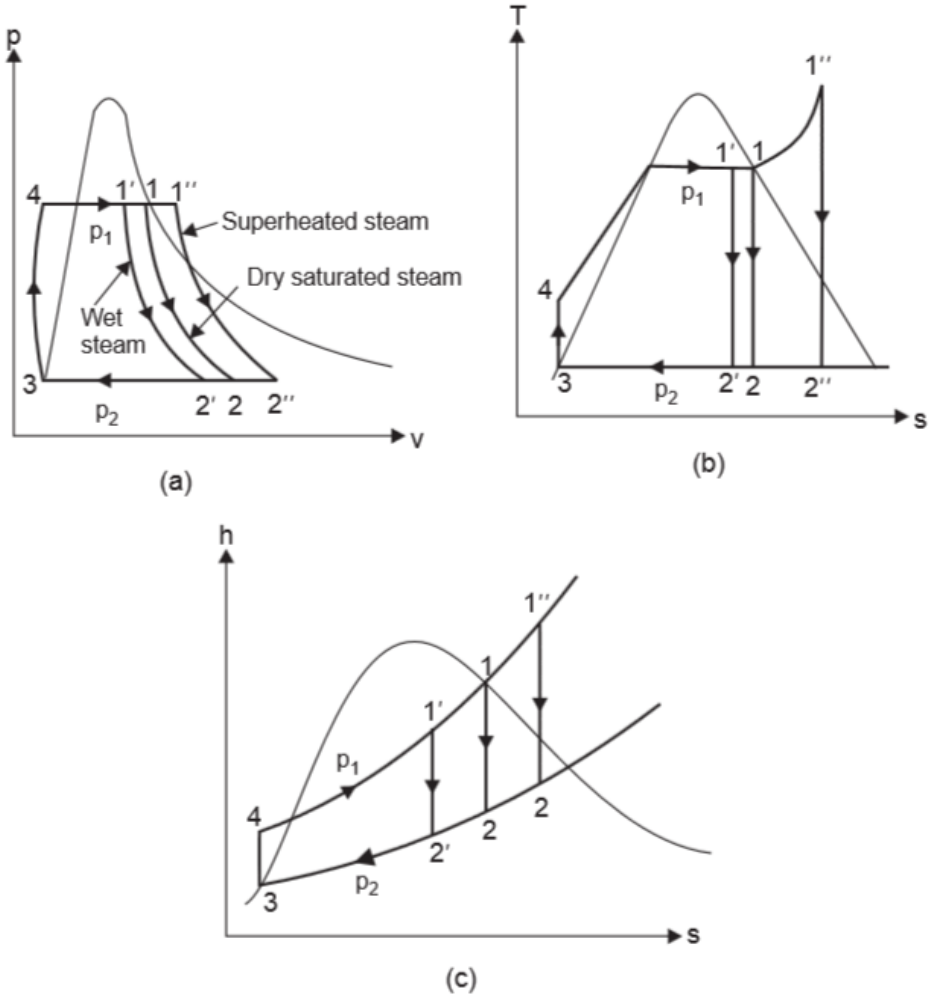
4. The cycle is still more difficult to operate in practice with superheated steam due to the necessity of supplying the superheat at constant temperature instead of constant pressure (as it is customary).

- In a practical cycle, limits of pressure and volume are far more easily realised than limits of temperature so that at present no practical engine operates on the Carnot cycle, although all modern cycles aspire to achieve it.

RANKINE CYCLE

Rankine cycle is the theoretical cycle on which the steam turbine (or engine) works





The Rankine cycle is shown in Fig.. It comprises of the following processes :

Process 1-2 : Reversible adiabatic expansion in the turbine (or steam engine).

Process 2-3 : Constant-pressure transfer of heat in the condenser.

Process 3-4 : Reversible adiabatic pumping process in the feed pump.

Process 4-1 : Constant-pressure transfer of heat in the boiler.

Fig. shows the Rankine cycle on p-v, T-s and h-s diagrams (when the saturated steam enters the turbine, the steam can be wet or superheated also).

Considering 1 kg of fluid :

Applying steady flow energy equation (S.F.E.E.) to boiler, turbine, condenser and pump :

(i) For boiler (as control volume), we get

$h_{f4} + Q_1 = h_1$

$\therefore Q_1 = h_1 - h_{f4}$

(ii) For turbine (as control volume), we get

$h_1 = W_T + h_2$, where W_T = turbine work

$\therefore W_T = h_1 - h_2$

(iii) For condenser, we get

$h_2 = Q_2 + h_{f3}$

$\therefore Q_2 = h_2 - h_{f3}$

(iv) For the feed pump, we get

$h_{f3} + W_P = h_{f4}$, where, W_P = Pump work

$\therefore W_P = h_{f4} - h_{f3}$

Now, efficiency of Rankine cycle is given by

$$\eta_{Rankine} = \frac{W_{net}}{Q_1} = \frac{W_T - W_P}{Q_1} = \frac{(h_1 - h_2) - (h_{f4} - h_{f3})}{(h_1 - h_{f4})}$$

The feed pump handles liquid water which is incompressible which means with the increase in pressure its density or specific volume undergoes a little change. Using general property relation for reversible adiabatic compression, we get

$Tds = dh - vdp$

$ds = 0$

$\therefore \quad dh = vdp$

or $\Delta h = v\,\Delta p$ ..... (since change in specific volume is negligible) or $h_{f4} - h_{f3} = v_3 (p_1 - p_2)$

When p is in bar and v is in m^3/kg, we have

$h_{f4} - h_{f3} = v_3 (p_1 - p_2) \times 10^5$ J/kg

The feed pump term ($h_{f4} - h_{f3}$) being a small quantity in comparison with turbine work, W_T, is usually neglected, especially when the boiler pressures are low.

$$Then, \eta_{Rankine} = \frac{(h_1 - h_2)}{(h_1 - h_{f4})}$$

Comparison between Rankine Cycle and Carnot Cycle

The following points are worth noting :

(i) Between the same temperature limits Rankine cycle provides a higher specific work output than a Carnot cycle, consequently Rankine cycle requires a smaller steam flow rate resulting in smaller size plant for a given power output. However, Rankine cycle calls for higher rates of heat transfer in boiler and condenser.

(ii) Since in Rankine cycle only part of the heat is supplied isothermally at constant higher temperature T1, therefore, its efficiency is lower than that of Carnot cycle. The efficiency of the Rankine cycle will approach that of the Carnot cycle more nearly if the superheat temperature rise is reduced.

(iii) The advantage of using pump to feed liquid to the boiler instead to compressing a wet vapour is obvious that the work for compression is very large compared to the pump.

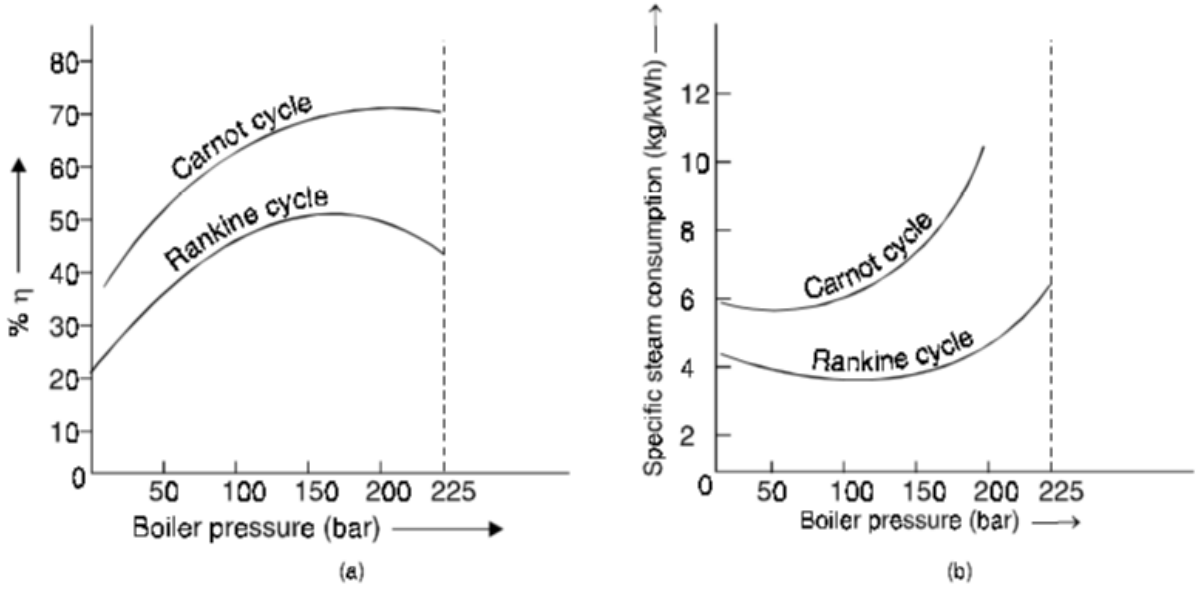
Fig. shows the plots between efficiency and specific steam consumption against boiler pressure for Carnot and ideal Rankine cycles.

Effect of Operating Conditions on Rankine Cycle Efficiency

The Rankine cycle efficiency can be improved by :

(i) Increasing the average temperature at which heat is supplied.

(ii) Decreasing/reducing the temperature at which heat is rejected

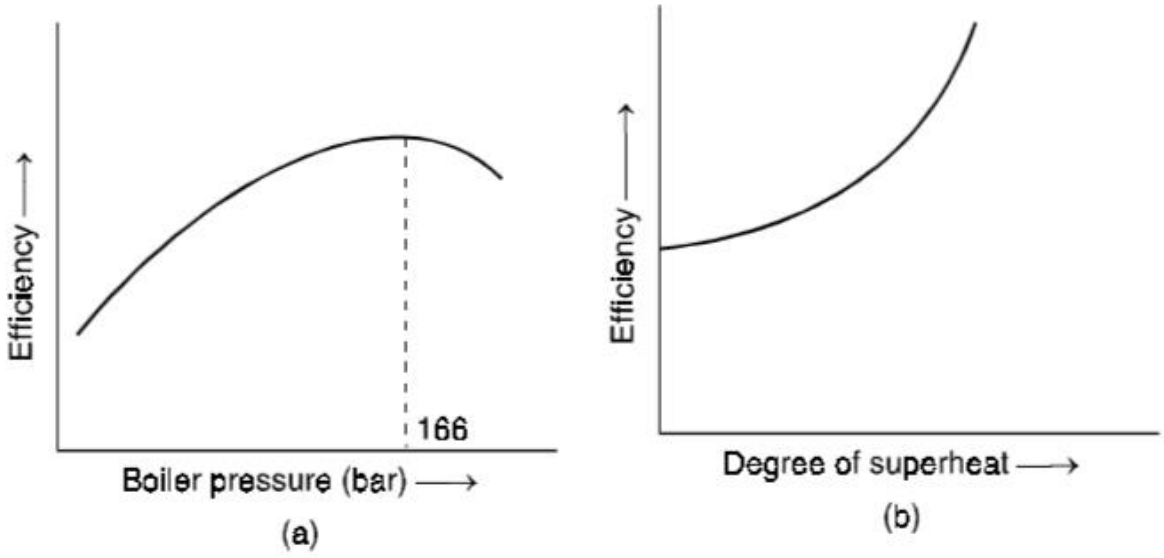


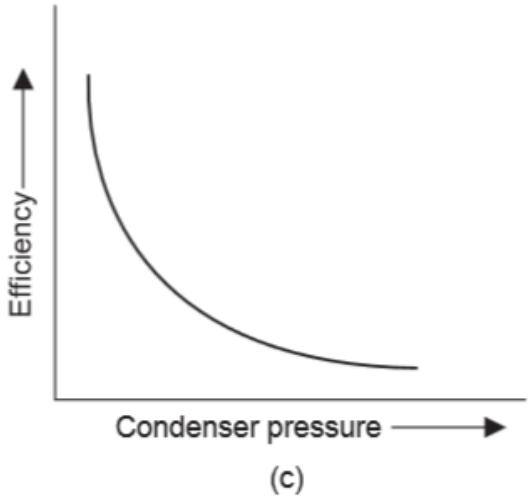
This can be achieved by making suitable changes in the conditions of steam generation or condensation, as discussed below :

1. Increasing boiler pressure. It has been observed that by increasing the boiler pressure (other factors remaining the same) the cycle tends to rise and reaches a maximum value at a boiler pressure of about 166 bar [Fig (a)].

2. Superheating. All other factors remaining the same, if the steam is superheated before allowing it to expand the Rankine cycle efficiency may be increased [Fig(b)]. The use of superheated steam also ensures longer turbine blade life because of the absence of erosion from high velocity water particles that are suspended in wet vapour.

3. Reducing condenser pressure. The thermal efficiency of the cycle can be amply improved by reducing the condenser pressure [Fig (c)] (hence by reducing the temperature at which heat is rejected), especially in high vacuums. But the increase in efficiency is obtained at the increased cost of condensation apparatus.





(c)

The thermal efficiency of the Rankine cycle is also improved by the following methods :

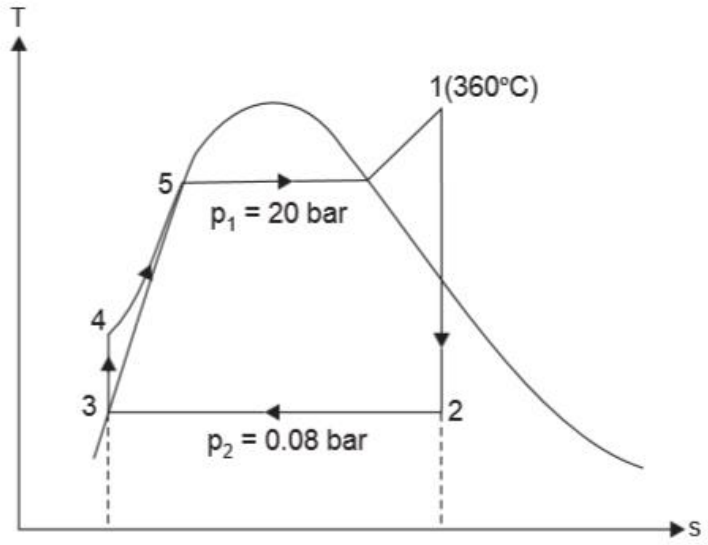
i) By regenerative feed heating.

(iii) By reheating of steam.

(iv) By water extraction.

(v) By using binary-vapour.

Example. In a steam turbine steam at 20 bar, 360°C is expanded to 0.08 bar. It then enters a condenser, where it is condensed to saturated liquid water. The pump feeds back the water into the boiler. Assume ideal processes, find per kg of steam the net work and the cycle efficiency.



Solution. Boiler pressure, p_1 = 20 bar (360°C)

Condenser pressure, p_2 = 0.08 bar

From steam tables :

At 20 bar (p_1), 360°C : h_1 = 3159.3 kJ/kg

s_1 = 6.9917 kJ/kg-K

At 0.08 bar (p_2) : $h_3 = h_{f\,(p2)}$ = 173.88 kJ/kg, $s_3 = s_{f\,(p2)}$ = 0.5926 kJ/kg-K

$h_{fg(\,p2)}$ = 2403.1 kJ/kg, $s_{g\,(p2)}$ = 8.2287 kJ/kg-K $v_{f(\,p2)}$ = 0.001008 m^3/kg ∴ $s_{fg\,(p2)}$ = 7.6361 kJ/kg-K

Now $s_1 = s_2$

$6.9917 = s_{f\,(p2)} + x_2\, s_{fg\,(p2)} = 0.5926 + x_2 \times 7.6361$

x2 = 0.838

∴ $h_2 = h_{f\,(p2)} + x_2\, h_{fg\,(p2)}$ = 173.88 + 0.838 × 2403.1 = 2187.68 kJ/kg.

Net work, W_{net} :

$W_{net} = W_{turbine} - W_{pump}$

$W_{pump} = h_{f4} - h_{f\,(p2)} (= h_{f3}) = v_{f\,(p2)}\,(p_1 - p_2)$

= 0.00108 (m3/kg) × (20 – 0.08) × 100 kN/m^2 = 2.008 kJ/kg

[and $h_{f4} = 2.008 + h_{f\,(p2)} = 2.008 + 173.88 = 175.89$ kJ/kg]

$W_{turbine} = h_1 - h_2 = 3159.3 - 2187.68 = 971.62$ kJ/kg

∴ $W_{net} = 971.62 - 2.008 = 969.61$ kJ/kg. (Ans.)

Cycle efficiency, η_{cycle} :

$Q_1 = h_1 - h_{f4} = 3159.3 - 175.89 = 2983.41$ kJ/kg

∴ $\eta_{cycle} = W_{net}/ Q_1 = 969.61/ 2983.41$

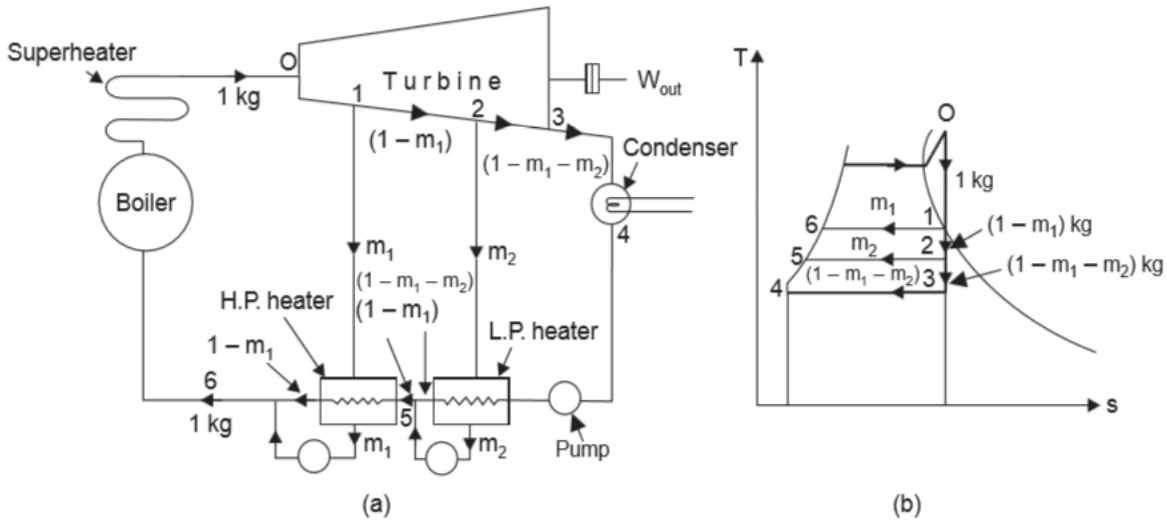
= 0.325 or 32.5%. (Ans.)

REGENERATIVE CYCLE

- In the Rankine cycle it is observed that the condensate which is fairly at low temperature has an irreversible mixing with hot boiler water and this results in decrease of cycle efficiency. Methods are, therefore, adopted to heat the feed water from the hot well of condenser irreversibly by interchange of heat within the system and thus improving the cycle efficiency. This heating method is called regenerative feed heat and the cycle is called regenerative cycle.

- The principle of regeneration can be practically utilised by extracting steam from the turbine at several locations and supplying it to the regenerative heaters. The resulting cycle is known as regenerative or bleeding cycle.

- Fig. (a) shows a diagrammatic layout of a condensing steam power plant in which a surface condenser is used to condense all the steam that is not extracted for feed water heating. The turbine is double extracting and the boiler is equipped with a superheater. The cycle diagram (T-s) would appear as shown in Fig. (b). This arrangement constitutes a regenerative cycle.



Let, m_1 = kg of high pressure (H.P.) steam per kg of steam flow,

m_2 = kg of low pressure (L.P.) steam extracted per kg of steam flow, and

$(1 - m_2 - m_2)$ = kg of steam entering condenser per kg of steam flow.

Energy/Heat balance equation for H.P. heater :

$m_1 (h_1 - h_{f6}) = (1 - m_1) (h_{f6} - h_{f5})$

or $m_1[(h_1 - h_{f6}) + (h_{f6} - h_{f5})] = (h_{f6} - h_{f5})$

$$or, m_1 = \frac{h_{f6} - h_{f5}}{h_1 - h_{f5}}$$

Energy/Heat balance equation for L.P. heater :

$m_2 (h_2 - h_{f5}) = (1 - m_1 - m_2) (h_{f5} - h_{f3})$

or $m_2[(h_2 - h_{f5}) + (h_{f5} - h_{f3})] = (1 - m_1) (h_{f5} - h_{f3})$

$$or, m_2 = \frac{(1-m_1)(h_{f5} - h_{f3})}{(h_2 - h_{f3})}$$

All enthalpies may be determined ; therefore m_1 and m_2 may be found. The maximum temperature to which the water can be heated is dictated by that of bled steam. The condensate from the bled steam is added to feed water.

Neglecting pump work :

The heat supplied externally in the cycle

$= (h_0 - h_{f6})$

Isentropic work done = $m_1 (h_0 - h_1) + m_2 (h_0 - h_2) + (1 - m_1 - m_2) (h_0 - h_3)$

The thermal efficiency of regenerative cycle is

$$\eta_{thermal} = \frac{work done}{heat \sup plied} = \frac{m_1(h_0 - h_1) + m_2(h_0 - h_2) + (1-m_1-m_2)(h_0 - h_3)}{(h_0 - h_{f6})}$$

The work done by the turbine may also be calculated by summing up the products of the steam flow and the corresponding heat drop in the turbine stages.

i.e., Work done = $(h_0 - h_1) + (1 - m_1) (h_1 - h_2) + (1 - m_1 - m_2) (h_2 - h_3)$

Advantages of Regenerative cycle over Simple Rankine cycle :

1. The heating process in the boiler tends to become reversible.

2. The thermal stresses set up in the boiler are minimised. This is due to the fact that temperature ranges in the boiler are reduced.

3. The thermal efficiency is improved because the average temperature of heat addition to the cycle is increased.

4. Heat rate is reduced.

5. The blade height is less due to the reduced amount of steam passed through the low pressure stages.

6. Due to many extractions there is an improvement in the turbine drainage and it reduces erosion due to moisture.

7. A small size condenser is required.

Disadvantages :

1. The plant becomes more complicated.

2. Because of addition of heaters greater maintenance is required.

3. For given power a large capacity boiler is required.

4. The heaters are costly and the gain in thermal efficiency is not much in comparison to the heavier costs.

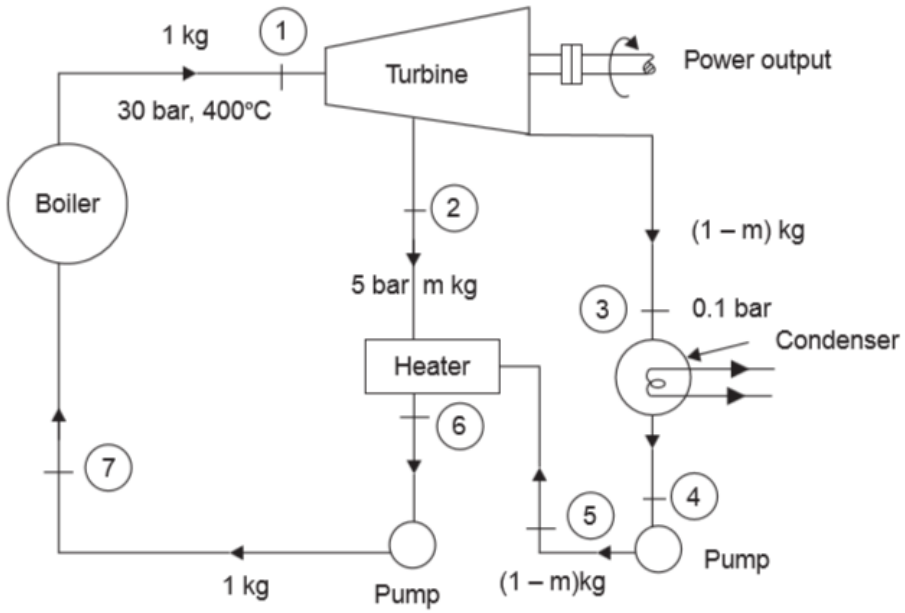
Example . In a single-heater regenerative cycle the steam enters the turbine at 30 bar, 400°C and the exhaust pressure is 0.10 bar. The feed water heater is a direct contact type which operates at 5 bar. Find :

(i) The efficiency and the steam rate of the cycle.

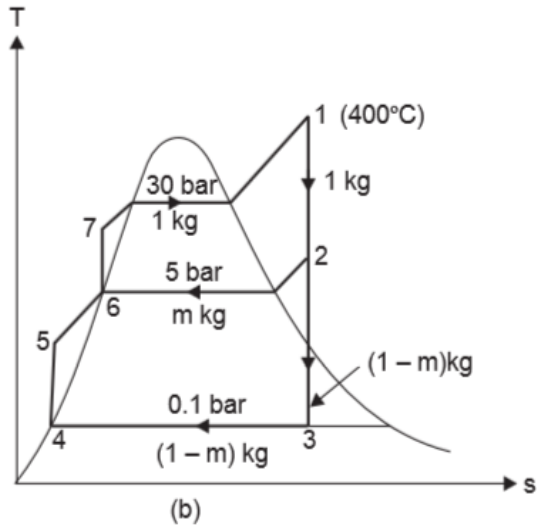
(ii) The increase in mean temperature of heat addition, efficiency and steam rate as compared to the Rankine cycle (without regeneration).

Pump work may be neglected

Solution. Fig. shows the flow, T-s and h-s diagrams.



(a)



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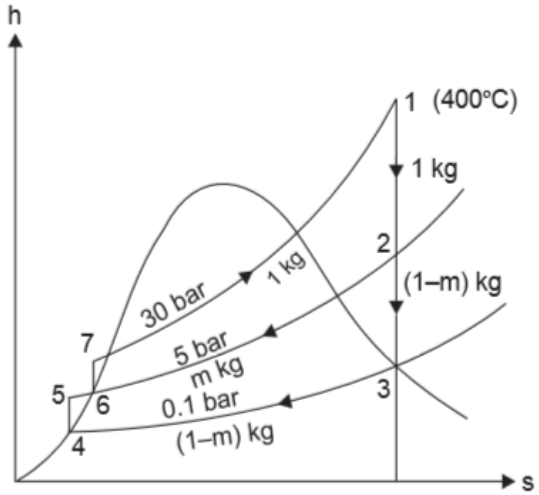


(c)

From steam tables :

At 30 bar, 400 ° C : $h_1 = 3230.9$ kJ/kg, $s_1 = 6.921$ kJ/kg K = $s_2 = s_3$,

At 5 bar : $s_f = 1.8604$, $s_g = 6.8192$ kJ/kg K, $h_f = 640.1$ kJ/kg

Since $s_2 > s_g$, the state 2 must lie in the superheated region. From the table for superheated steam $t_2 = 172°C$, $h_2 = 2796$ kJ/kg.

At 0.1 bar : $s_f = 0.649$, $s_{fg} = 7.501$, $h_f = 191.8$, $h_{fg} = 2392.8$

Now, $s_2 = s_3$

i.e., $6.921 = s_{f3} + x_3 s_{fg3} = 0.649 + x_3 \times 7.501$

$$\therefore x_3 = \frac{6.921 - 0.649}{7.501} = 0.836$$

$\therefore$ $h_3 = h_{f3} + x_3 h_{fg3} = 191.8 + 0.836 \times 2392.8 = 2192.2$ kJ/kg

Since pump work is neglected

$h_{f4} = 191.8$ kJ/kg = h_{f5}

$h_{f6} = 640.1$ kJ/kg (at 5 bar) = h_{f7}

Energy balance for heater gives $m (h_2 - h_{f6}) = (1 - m) (h_{f6} - h_{f5})$

$m (2796 - 640.1) = (1 - m) (640.1 - 191.8) = 448.3 (1 - m)$

$2155.9\ m = 448.3 - 448.3\ m$

∴ m = 0.172 kg

∴ Turbine work, $W_T = (h_1 - h_2) + (1 - m)(h_2 - h_3)$

= (3230.9 – 2796) + (1 – 0.172) (2796 – 2192.2)

= 434.9 + 499.9 = 934.8 kJ/kg

Heat supplied, $Q_1 = h_1 - h_{f6}$ = 3230.9 – 640.1 = 2590.8 kJ/kg.

(i) Efficiency of cycle, η_{cycle} :

$$\eta_{cycle} = \frac{W_T}{Q_1} = \frac{934.8}{2590.8} = 0.3608 = 36.08\%$$

$$Steamrate = \frac{3600}{934.8} = 3.85 kg/kWh$$

$$T_{m_1} = \frac{h_1 - h_{f7}}{s_1 - s_7} = \frac{2590.8}{6.921 - 1.8604} = 511.9K = 238.9^0 C$$

(ii)

$$T_{m_1}(without regeneration) = \frac{h_1 - h_{f4}}{s_1 - s_4} = \frac{3230.9 - 191.8}{6.921 - 0.649} = 484.5K = 211.5^0 C$$

Increase in T_{m1} due to regeneration

= 238.9 – 211.5 = 27.4°C. (Ans.)

W_T (without regeneration) = $h_1 - h_3$ = 3230.9 – 2192.2 = 1038.7 kJ/kg

Steam rate without regeneration

= 3600/ 1038.7 = 3.46 kg/kWh

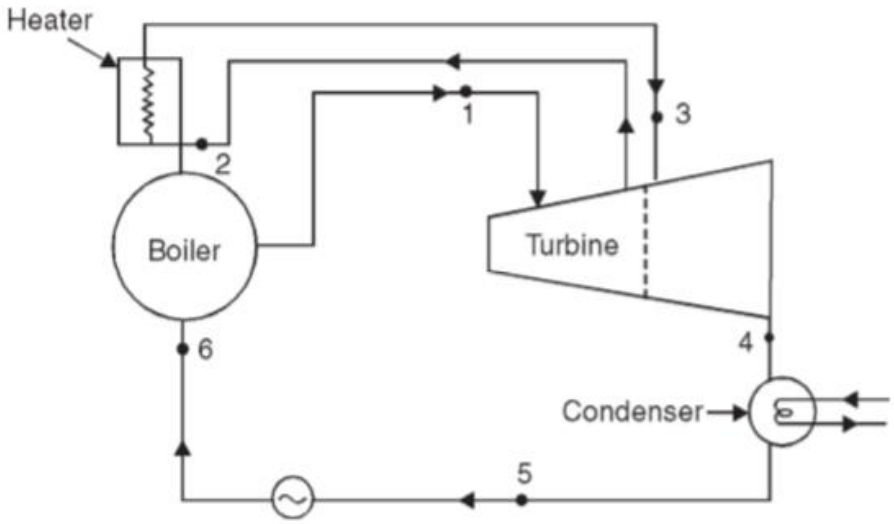
∴ Increase in steam rate due to regeneration = 3.85 – 3.46 = 0.39 kg/kWh. (Ans.)

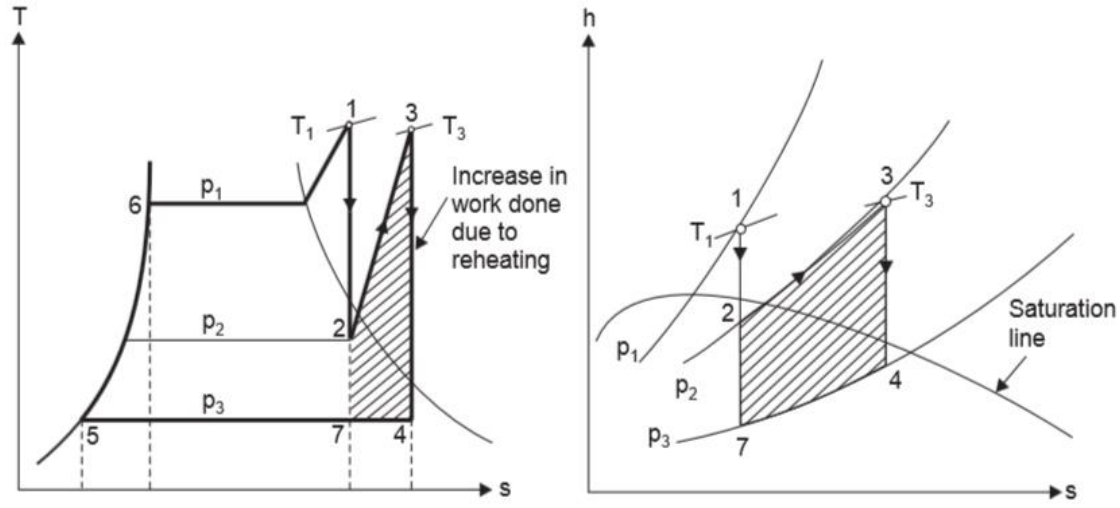
$$\eta_{cycle(without regeneration)} = \frac{h_1 - h_3}{h_1 - h_{f4}} = \frac{1038.7}{3230.9 - 191.8} = 0.3418 = 34.18\%$$

Increase in cycle efficiency due to regeneration = 36.08 – 34.18 = 1.9%. (Ans.)

REHEAT CYCLE

- Fig. shows a schematic diagram of a theoretical single-stage reheat cycle. The corresponding representation of ideal reheating process on T-s and h-s chart is shown in Figs. (a and b).





- In fig. 5-1 shows the formation of steam in the boiler. The steam as at state point 1 (i.e., pressure p1 and temperature T1) enters the turbine and expands isentropically to a certain pressure p2 and temperature T2.

- From this state point 2 the whole of steam is drawn out of the turbine and is reheated in a reheater to a temperature T3. (Although there is an optimum pressure at which the steam should be removed for reheating, if the highest return is to be obtained, yet, for simplicity, the

whole steam is removed from the high pressure exhaust, where the pressure is about one-fifth of boiler pressure, and after undergoing a 10% pressure drop, in circulating through the heater, it is returned to intermediate pressure or low pressure turbine). This reheated steam is then readmitted to the turbine where it is expanded to condenser pressure isentropically.

Note. Superheating of steam. The primary object of superheating steam and supplying it to the primemovers is to avoid too much wetness at the end of expansion. Use of inadequate degree of superheat in steam engines would cause greater condensation in the engine cylinder ; while in case of turbines the moisture content of steam would result in undue blade erosion. The maximum wetness in the final condition of steam that may be tolerated without any appreciable harm to the turbine blades is about 12 per cent. Broadly each 1 per cent of moisture in steam reduces the efficiency of that part of the turbine in which wet steam passes by 1 per cent to 1.5 per cent and in engines about 2 per cent.

Thermal efficiency with 'Reheating' (neglecting pump work) :

Heat supplied = $(h_1 - h_{f4}) + (h_3 - h_2)$

Heat rejected = $h_4 - h_{f4}$

Work done by the turbine = Heat supplied – heat rejected

$= (h_1 - h_{f4}) + (h_3 - h_2) - (h_4 - h_{f4})$

$= (h_1 - h_2) + (h_3 - h_4)$

Thus, theoretical thermal efficiency of reheat cycle is

$$\eta_{thermal} = \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_{f4}) + (h_3 - h_2)}$$

Thermal efficiency without reheating is

$$\eta_{thermal} = \frac{h_1 - h_7}{h_1 - h_{f4}} (\because h_{f4} = h_7)$$

Advantages of 'Reheating' :

1. There is an increased output of the turbine.

2. Erosion and corrosion problems in the steam turbine are eliminated/avoided.

3. There is an improvement in the thermal efficiency of the turbines.

4. Final dryness fraction of steam is improved.

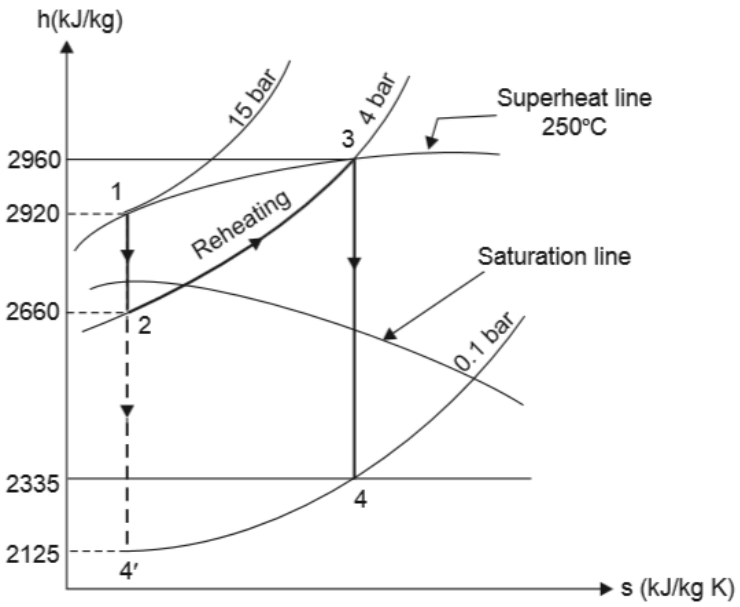
5. There is an increase in the nozzle and blade efficiencies.

Disadvantages :

1. Reheating requires more maintenance.

2. The increase in thermal efficiency is not appreciable in comparison to the expenditure incurred in reheating.

Example. Steam at a pressure of 15 bar and 250°C is expanded through a turbine at first to a pressure of 4 bar. It is then reheated at constant pressure to the initial temperature of 250°C and is finally expanded to 0.1 bar. Using Mollier chart, estimate the work done per kg of steam flowing through the turbine and amount of heat supplied during the process of reheat. Compare the work output when the expansion is direct from 15 bar to 0.1 bar without any reheat. Assume all expansion processes to be isentropic.



Pressure, p1 = 15 bar ;

p2 = 4 bar ;

p4 = 0.1 bar.

Work done per kg of steam,

W = Total heat drop

= $[(h_1 - h_2) + (h_3 - h_4)]$ kJ/kg .....(i)

Amount of heat supplied during process of reheat,

$h_{reheat} = (h_3 – h_2)$ kJ/kg ...(ii)

From Mollier diagram or h-s chart,

$h_1 = 2920$ kJ/kg, $h_2 = 2660$ kJ/kg

$h_3 = 2960$ kJ/kg, $h_4 = 2335$ kJ/kg

Now, by putting the values in eqns. (i) and (ii), we get

$W = (2920 – 2660) + (2960 – 2335)$

= 885 kJ/kg. (Ans.)

Hence work done per kg of steam = 885 kJ/kg. (Ans.)

Amount of heat supplied during reheat,

$h_{reheat} = (2960 – 2660) = 300$ kJ/kg. (Ans.)

If the expansion would have been continuous without reheating i.e., 1 to 4', the work output is given by

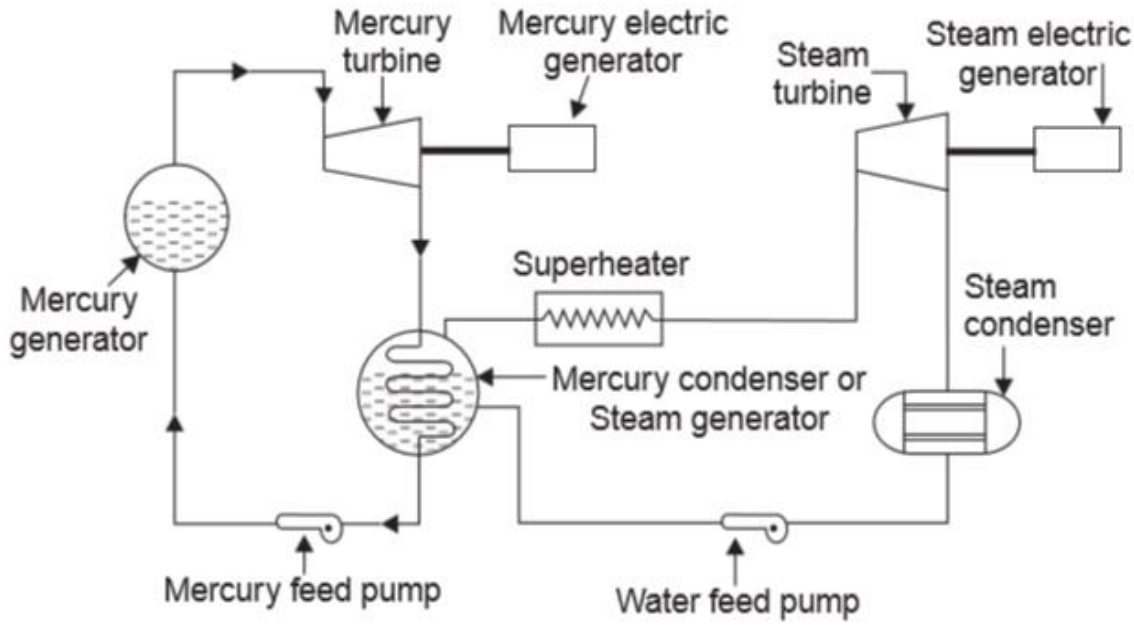
$W_1 = h_1 – h_4'$

From Mollier diagram, $h_4' = 2125$ kJ/kg

$\therefore$ $W_1 = 2920 – 2125 = 795$ kJ/kg. (Ans.)

BINARY VAPOUR CYCLE

- Fig. shows the schematic line diagram of binary vapour cycle using mercury and water as working fluids. The processes are represented on T-s diagram are also shown in Fig.



Analysis of Binary vapour cycle :

h_{hg1} = Heat supplied per kg of Hg (mercury) vapour formed in the mercury boiler.

h_{hg2} = Heat lost by one kg of Hg vapour in the mercury condenser.

h_s = Heat given per kg of steam generated in the mercury condenser or steam boiler.

W_{hg} = Work done per kg of Hg in the cycle.

W_s = Work done per kg of steam in the steam cycle.

η_s = Thermal efficiency of the steam cycle.

η_{hg} = Thermal efficiency of the Hg cycle.

m = Mass of Hg in the Hg cycle per kg of steam circulated in the steam cycle.

The heat losses to the surroundings, in the following analysis, are neglected and steam generated is considered one kg and Hg in the circuit is m kg per kg of water in the steam cycle.

Heat supplied in the Hg boiler $h_t = m \times h_{hg1}$

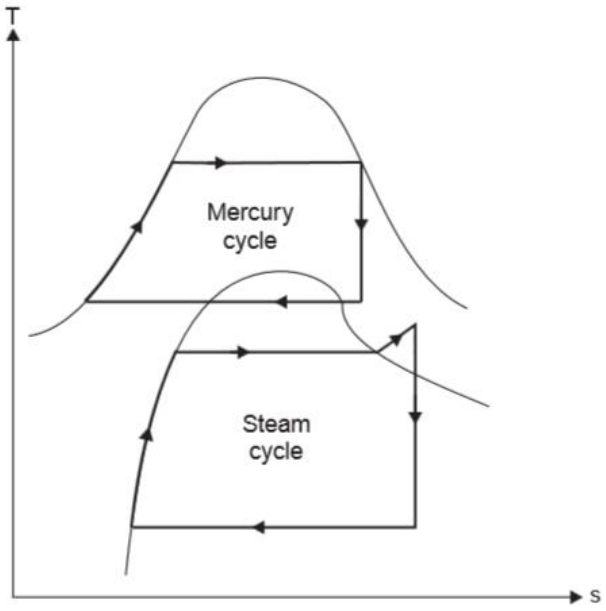
Work done in the mercury cycle = $m \cdot W_{hg}$

Work done in the steam cycle= $1 \times W_s$

Total work done in the binary cycle is given by $W_t = m\,W_{hg} + W_s$

∴ Overall efficiency of the binary cycle is given by

$$\eta = \frac{workdone}{heat\,\text{sup}\,plied} = \frac{W_t}{h_t} = \frac{mW_{hg} + W_s}{mh_{hg1}}$$



Thermal efficiency of the mercury cycle is given by

$$\eta_{hg} = \frac{mW_{hg}}{mh_{hg1}} = \frac{W_{hg}}{h_{hg1}} = \frac{h_{hg1} - h_{hg2}}{h_{hg1}} = 1 - \frac{h_{hg2}}{h_{hg1}} = \frac{mh_{hg1} - h_s}{mh_{hg1}} = 1 - \frac{1}{m}\frac{h_s}{h_{hg1}}$$

Heat lost by mercury vapour = Heat gained by steam

∴ $mh_{hg2} = 1 \times h_s$

Substituting the value of m, we get

$$\eta_{hg} = 1 - \frac{h_{hg2}}{h_{hg1}}$$

The thermal efficiency of the steam cycle is given by

$$\eta_s = \frac{W_s}{h_s} = \frac{h_{s2} - h_{s1}}{h_{s1}} = \frac{h_{s1} - h_{s2}}{mh_{hg2}}$$

By combining above equations we get

$\eta = \eta_{hg} (1 - \eta_s) + \eta_s$

Example: A binary vapour cycle operates on mercury and steam. Standard mercury vapour at 4.5 bar is supplied to the mercury turbine, from which it exhausts at 0.04 bar. The mercury condenser generates saturated steam at 15 bar which is expanded in a steam turbine to 0.04 bar.

(i) Determine the overall efficiency of the cycle.

(ii) If 48000 kg/h of steam flows through the steam turbine, what is the flow through the mercury turbine ?

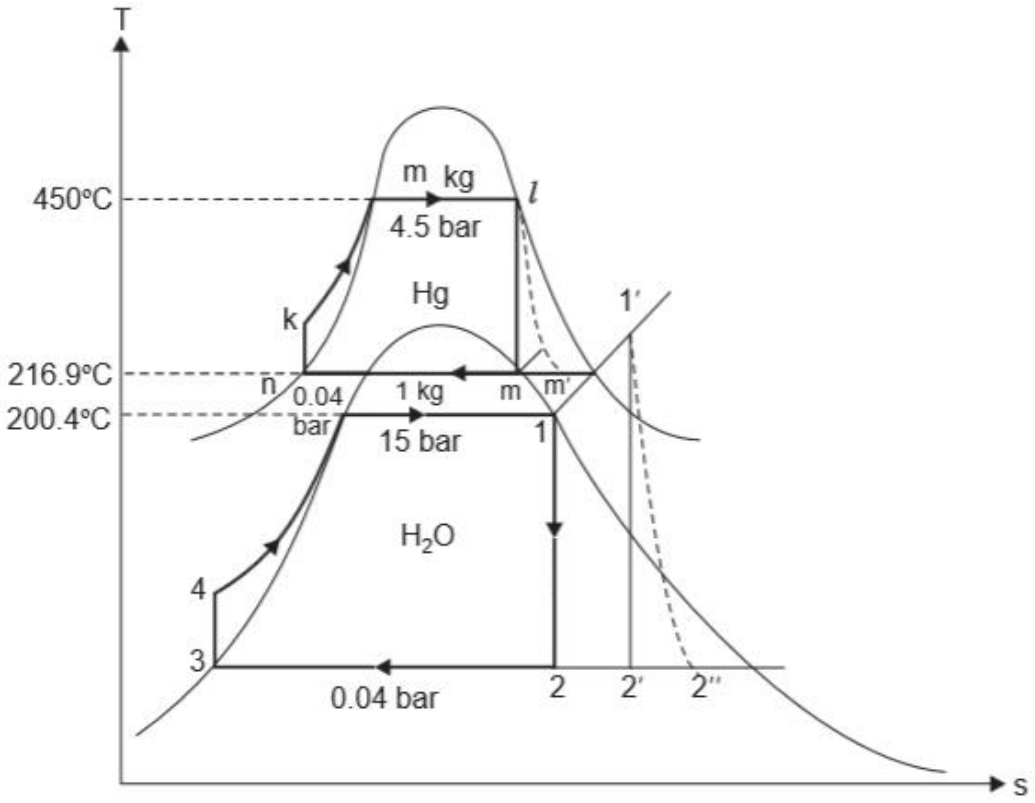
(iii) Assuming that all processes are reversible, what is the useful work done in the binary vapour cycle for the specified steam flow ?

(iv) If the steam leaving the mercury condenser is superheated to a temperature of 300°C in a superheater located in the mercury boiler and if the internal efficiencies of the mercury and steam turbines are 0.84 and 0.88 respectively, calculate the overall efficiency of the cycle.

The properties of standard mercury are given below :

p (bar)	t (°C)	h_f(kJ/kg)	h_g(kJ/kg)	s_f(kJ/kg K)	s_g(kJ/kgK)	v_f (m^3/kg)	v_g (m^3/kg)
4.5	450	62.93	355.98	0.1352	0.5397	79.9×10^{-6}	0.068
0.04	216.9	29.98	329.85	0.0808	0.6925	76.5×10^{-6}	5.178.

Solution. The binary vapour cycle is shown in Fig.



Mercury cycle :

h_l = 355.98 kJ/kg

$s_l = 0.5397 = s_m = s_f + x_m s_{fg}$ or $0.5397 = 0.0808 + x_m (0.6925 - 0.0808)$

$$\therefore x_m = \frac{(0.5397 - 0.0808)}{(0.6925 - 0.0808)} = 0.75$$

$h_m = h_f + x_m h_{fg} = 29.98 + 0.75 \times (329.85 - 29.98)$

= 254.88 kJ/kg

Work obtained from mercury turbine

$(W_T)_{Hg} = h_l - h_m = 355.98 - 254.88 = 101.1$ kJ/kg

Pump work in mercury cycle,

$(W_P)_{Hg} = h_{fk} - h_{fn} = 76.5 \times 10{-}6 \times (4.5 - 0.04) \times 100 = 0.0341$ kJ/kg

$\therefore$ $W_{net} = 101.1 - 0.0341 = 101.1$ kJ/kg

$Q_1 = h_l - h_{fk} = 355.98 - 29.98 = 326$ kJ/kg

$(\because h_{fn} = h_{fk})$

$$\therefore \eta_{Hgcycle} = \frac{W_{net}}{Q_1} = \frac{101.1}{326} = 0.31 or 31\%$$

Steam cycle :

At 15 bar : $h_1 = 2789.9$ kJ/kg, $s_1 = 6.4406$ kJ/kg

At 0.04 bar : $h_f = 121.5$ kJ/kg, $h_{fg} = 2432.9$ kJ/kg,

$s_f = 0.432$ kJ/kg K, $s_{fg2} = 8.052$ kJ/kg K, $v_f = 0.0001$ kJ/kg K

Now, s1 = s2

$6.4406 = s_f + x_2 s_{fg} = 0.423 + x_2 \times 8.052$

$$\therefore x_2 = \frac{6.4406 - 0.423}{8.052} = 0.747$$

$h_2 = h_{f2} + x_2 h_{fg} = 121.5 + 0.747 \times 2432.9 = 1938.8$ kJ/kg

Work obtained from steam turbine,

$(W_T)_{steam} = h_1 - h_2 = 2789.9 - 1938.8 = 851.1$ kJ/kg

Pump work in steam cycle,

$(W_P)_{steam} = h_{f4} - h_{f3} = 0.001\ (15 - 0.04) \times 100 = 1.496$ kJ/kg = 1.5 kJ/kg

or $h_{f4} = h_{f3} + 1.5 = 121.5 + 1.5 = 123$ kJ/kg

$Q_1 = h_1 - h_{f4} = 2789.9 - 123 = 2666.9$ kJ/kg

$(W_{net})_{steam} = 851.1 - 1.5 = 849.6$ kJ/kg

$$\therefore \eta_{steamcycle} = \frac{W_{net}}{Q_1} = \frac{849.6}{2666.6} = 0.318 = 31.8\%$$

(i) Overall efficiency of the binary cycle :

Overall efficiency of the binary cycle

$= \eta_{Hgcycle} + \eta_{steam\ cycle} - \eta_{Hgcycle} \times \eta_{steam\ cycle}$

= 0.31 + 0.318 – 0.31 × 0.318 = 0.5294 or 52.94%

Hence overall efficiency of the binary cycle = 52.94%. (Ans.)

$\eta_{overall}$ can also be found out as follows :

Energy balance for a mercury condenser-steam boiler gives :

$m\,(h_m - h_{fn}) = 1(h_1 - h_{f4})$

where m is the amount of mercury circulating for 1 kg of steam in the bottom cycle

$$\therefore m = \frac{h_1 - h_{f4}}{h_m - h_{fn}} = \frac{2666.9}{254.88 - 29.98} = 11.86kg$$

$(Q_1)_{total} = m\,(h_l - h_{fk}) = 11.86 \times 326 = 3866.36$ kJ/kg

$(W_T)_{total} = m\,(h_l - h_m) + (h_1 - h_2)$

$= 11.86 \times 101.1 + 851.1 = 2050.1$ kJ/kg

$(W_P)_{total}$ may be neglected

$$\eta_{overall} = \frac{W_T}{Q_1} = \frac{2050.1}{3866.36} = 0.53 = 53\%$$

(ii) Flow through mercury turbine :

If 48000 kg/h of steam flows through the steam turbine, the flow rate of mercury,

$m_{Hg} = 48000 \times 11.86 = 569280$ kg/h. (Ans.)

(iii) Useful work in binary vapour cycle :

Useful work, $(W_T)_{total} = 2050.1 \times 48000 = 9840.5 \times 10^4$ kJ/h

$$= \frac{9840.5X10^4}{3600} = 27334.7KW = 27.33MW$$

(iv) Overall efficiency under new conditions :

Considering the efficiencies of turbines, we have :

$(W_T)_{Hg} = h_l - h_m' = 0.84 \times 101.1 = 84.92$ kJ/kg

$\therefore h_m' = h_l - 84.92 = 355.98 - 84.92 = 271.06$ kJ/kg

$\therefore$ $m' (h_m' - h_n') = (h_1 - h_{f4})$

$$orm' = \frac{h_1 - h_{f4}}{h_m' - h_n'} = \frac{2666.9}{271.06 - 29.98} = 11.06kg$$

$(Q_1)_{total} = m' (h_l - h_{fk}) + 1 (h_1' - h_1)$

[At 15 bar, 300°C : $h_g = 3037.6$ kJ/kg, $s_g = 6.918$ kJ/kg K]

$= 11.06 \times 326 + (3037.6 - 2789.9) = 3853.26$ kJ/kg

$s_1' = 6.918 = s_2' = 0.423 + x_2' \times 8.052$

$x_2' = 0.80$

$h_2' = 121.5 + 0.807 \times 2432.9 = 2084.8$ kJ/kg

$(W_T)_{steam} = h_1' - h_2' = 0.88 (3037.6 - 2084.8)$

$= 838.46$ kJ/kg

$(W_T)_{total} = 11.06 \times 84.92 + 838.46 = 1777.67$ kJ/kg

Neglecting pump work,

$$\eta_{overall} = \frac{1777.67}{3853.26} = 0.4613 = 46.13\%$$